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Optimal PMU Placement in 14 Bus System using Integer Linear Programming

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Abstract: The Phasor Measurement Unit (PMU) is a key device for monitoring and control of the power system network. It gives real time, synchronized measurements of voltages at the buses and also current phase values which are incident to those buses where these PMUs are located. This paper presents atopological approach to determine the optimal PMU placement in order to make the system completely observable usinginteger linear programming. The proposed formulation is tested on IEEE 14 test systems and results so obtained for complete observability of system at normal condition is presented in this paper. The proposed PMU placement has been implemented on IEEE 14 bus systems.

Keywords: Phasor Measurement Unit, Optimal PMU Placement, Integer Linear Programming.

I. INTRODUCTION

PMU is a measuring device used to measure voltage and current phasors. PMUs use a global positioning system (GPS) to provide synchronized measurements of real time phasors of voltage and currents [1]. A power system network is said to be observable when voltage phasors at all the buses are known viz. Obviously, when PMUs are installed at all the buses of network, and the measurements of all PMUs are communicated to control centers, then voltage phasors at all buses would be known. This equipment cans potentially revolution the traditional state estimation to state measurement [2]. However, as described later in this section, even when PMUs are not installed at all the buses, a power system network may be observable if the voltage phasors at the buses without PMUs can be calculated using the network parameters and the PMU measurements at other buses.Such buses are said to have pseudo-measurements [3].A number of PMUs are already installed in several utilities around the world for various applications such as adaptive protection, system protection schemes, and state estimation [1]. Other application areas include wide are a monitoring and control (WAMC), stability monitoring, and enhanced as well as efficient system utilization. PMUs are an integral part of smart grids, and hence the rate ofPMU installations is increasing. One of the mostimportant matters that need to be spoken in thedeveloping technology of PMUs is their placement. The important factor limiting the number of PMU installations is their priceand existing communication services, the cost of which may be higher than that of the PMUs [1]. So the communication and cost constraints of PMUs have motivated power engineers and researchers to find minimal PMU placement for intended applications.Several techniques and algorithms have been proposed for optimal PMU placement for power system observability during normal operating conditions [1], [3]-[6]. Optimal PMU placement for complete and incomplete observability has been proposed in [1] using spanning trees of a power system graph. An integer programming based method for optimal placement of PMUs is proposed in [4-5] for complete observability of a power system. The proposed integer programming formulation has considered system cases with normal condition. A generalized integer linear programming (ILP) method for optimal PMU placement has been discussed in [6-9] which includes cases of full observability, incomplete observability and placement of a PMU for redundancy [10]. Various aspects of the optimal PMU placement problem, and have proposed a procedure for multi-staging of PMU placement in a given time horizon using ILP have discussed in [11]. Multi-stage scheduling determines the locations of PMU placement in different stages of time horizon, and would be useful when utilities have cost constraint to place all the required PMUs in a single project. Thus, several methods and algorithms have been proposed for optimal PMU placement. The paper is arranged in IV sections, section I covers the introduction part. Section IIexplains the basicprinciples forobservability of the system. Section III explains OPP problem formulation and section IV present the simulation results, comparison and implementation of the PMU in different practical systems.

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Lastlysections V, VI covers the conclusion and references respectively.

II. OBSERVABILITY RULES

PMU is a dynamic device which measures the phase value of voltage where it is installed and also measures the phase value of current which are connected to that branchGPS time stamped measurement signals are fed to a Phasor Data Concentrator (PDC) by using PMUs. The PDC collects and sorts the phasor measurements and signal processor converts data of PMUs into useful information which is visible on Human Machine Interface (HMI).The operator can easily access the critical information of the power system state.Some assumptions can be formulated for the placement of PMUs which are:

Case 1: For PMU installed buses, voltage phasor and current phasor of all its incident branches are known. These are called 'direct measurements'. According to the function of the phasor measurement unit, a PMU located in the Bus D, as shown in Fig.1,indicates that the voltage in this bus can be directly measured

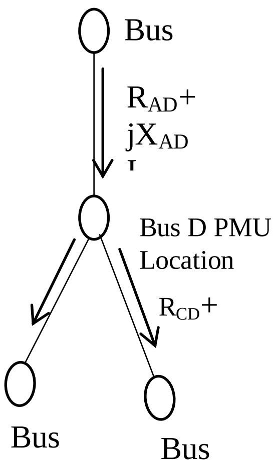

Fig.1 First Observability Rule

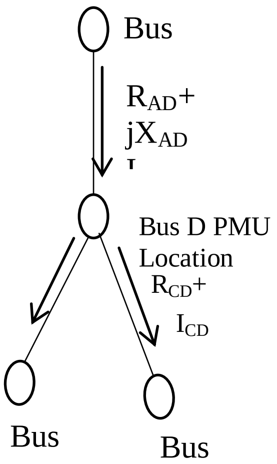

Fig.2 Second Observability Rule

Meanwhile, the branch currents attached to the node are also measured by the PMU. In Case 1, the known parameters measured by PMU are V_D, I_{AD}, I_{BD} and I_{CD} and the characteristic of transmission line are R_{AD}+ jX_{AD}, R_{BD}+ jX_{BD}, R_{CD}+ jX_{CD}.

Case 2: If the voltage and current phasors at one end of a branch are known, the voltage phasor at the other end of the branch can be obtained using equation 1 to 3. These are called 'pseudo measurements'.

Based on the known parameter such as line impedance and branch currents, the magnitude of voltage will be resolved using the following equation:

$$V_A = V_D + I_{AD} (R_{AD}+ jX_{AD}) \quad (1)$$
$$V_B = V_D - I_{BD} (R_{BD}+ jX_{BD}) \quad (2)$$
$$V_C = V_D - I_{CD} (R_{CD}+ jX_{CD}) \quad (3)$$

Case 3: If voltage phasors of both ends of a branch are known, the current phasor of this branch can be obtained directly. These measurements are also known as 'pseudo measurements'.

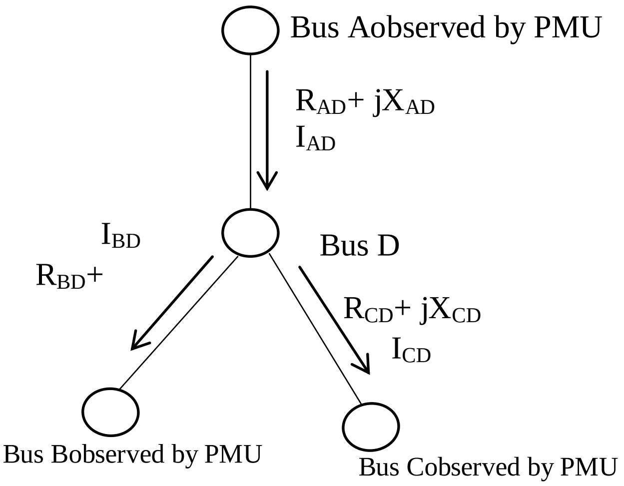

Fig.3 Third Observability Rule

Under this circumstance, assuming that the magnitudes of voltage in Bus A, Bus B and Bus Care observed and measured by the PMUs, the line current in the branch of BD, AD and CD as well as the voltage in Bus D can be calculated. The equations in solving the unknown information are as follows:

$$V_D = V_A - I_{AD} (R_{AD}+ jX_{AD}) \quad (4)$$
$$V_D = V_B + I_{BD} (R_{BD}+ jX_{BD}) \quad (5)$$
$$V_D = V_C + I_{CD} (R_{CD}+ jX_{CD}) \quad (6)$$
$$I_{AD} = I_{BD} + I_{CD} \quad (7)$$

The PMUs can be placed at planned buses to completely observe the total network. These located PMUs are measuring the voltage phase value of that bus and current phase values of the lines which are connected to the same bus. The aim is to completely observe the network with an optimum number of PMUs.

III. OPP PROBLEM FORMULATION

Integer programing is a mathematical programming method for solving an optimization problem which has the entire design variable as integer value is called integer programming. The objective function and the constraints are linear, nonlinear, or quadratic, thus leading to integer linear programming (ILP), integer nonlinear programing (INLP), and integer quadratic programming (IQP) algorithms, respectively.

The objective of PMU placement problem is to find minimum number of locations from where whole system becomes topologically observable. The problem is formulated as follows:

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Minimize $\sum_{j \in I} Uj$ (8)

Subject to $fi \geq 1, \forall i \in I$ (9)

Where, $fi = \sum_{j \in I} aijUi, \ \forall i \in I$ (10)

Using this formulation optimal PMU placement solution is obtained for a 9 bus system shown here.

Where,

i and j = indices of bus

I = set of buses

u_j= binary decision variable that is equal to 1 if PMU is installed at bus j and 0 otherwise

f_i= observability function of bus i

a_{ij}= binary connectivity parameter between bus i and j

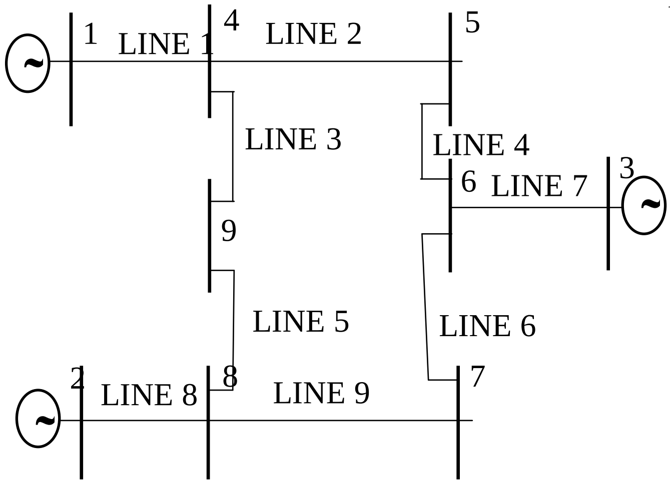

Fig4 Bus System

Also,

a_{ij}= 1, if i=j

=1, if i and j are connected

=0, otherwise

The observability of a bus depends on the installation of PMU on that bus or on one of its incident bus [2].Consider a nine bus system shown here and following the proposed formulation. So according to the proposed formulation the objective function and constraints will be as follows.

Minimize, $u1+u_2+u_3+u_4+u_5+u_6+u_7+u_8+u_9$

Subject to $f_1 = u_1+u_4>=1$

$f_2 = u_2+u_8>=1$

$f_3 = u_3+u_6>=1$

$f_4 = u_1+u_4+u_5+u_9>=1$

$f_5 = u_4+u_5+u_6>=1$

$f_6 = u_3+u_5+u_6+u_7>=1$

$f_7 = u_6+u_7+u_8>=1$

$f_8 = u_2+u_7+u_8+u_9>=1$

$f_9 = u_4+u_8+u_9>=1$

Using integer linear programming the optimal solution i.e. optimal location in given network where we can place a PMU and make all buses observable are u_2, u_4 and u_6. Now referring Fig.4, if we place a PMU on bus 2, bus 8 will also become observable as it is incident to bus 2. Similarly if a PMU is placed on bus 4 than, the three incident buses b_1 , b_5 and b_9 will also be observable and placing a PMU on bus 6 the incident buses to bus 6 which are b_3, b_5 and b_7 become observable. Hence in a nine bus system we do not have to put nine PMU devices to collect phasor data from those buses instead placing 3 PMU we can monitor whole system.

IV SIMULATION AND RESULT

By the topological information of a power system the interconnection of the various buses can be grouped in an array called node-incidence matrix. For example, for the IEEE 14-bus system,

A=

0	1	0	0	1	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	0	0	0	0	0
0	1	1	0	1	0	1	0	1	0	0	0	0	0
1	1	0	1	0	1	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	1	1	1	0
0	0	0	1	0	0	0	1	1	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	1	0	0	1	0	0	1	0	0	0	1
0	0	0	0	0	0	0	0	1	0	1	0	0	0
0	0	0	0	0	1	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	1	0
0	0	0	0	0	1	0	0	0	0	0	1	0	1
0	0	0	0	0	0	0	0	1	0	0	0	1	0

It is easy to find out that there are 8 nodes with degree 3 or more. Here K=8 and n=14. so, the number of PMUs needed that is ,the only possible values for S between 3 and 4 using equation (1) and (2) .Now the task is to finding out the minimum number of PMUs, and the dominating set S. The basic idea of this algorithm is to test all possible node combinations by the observation rules, until one combination is found to be able to "observe" all the system. A test is called for a combination as a measurement. For the IEEE 14-bus system, the maximum number of measurements is number of combinations produce by selecting numbers of a group in between 3-4, who will converge, will give the required number of PMUs in the system. It should be kept in mind that, in the implementation of the algorithm, it is not necessary to run all the measurements to find out the S-set. The number of measurement before we get an S-set (which is usually not unique) can be any number.

ALGORITHM FOR FINDING THE MINIMUM NUMBER OF PMUS

1. Start
2. Read in node-incidence matrix A with all buses (nodes) in the system says G.
3. Calculate the bounds of S.

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4. Check the observability of the system by creating loop starting from the lower bound, to the upper bound:
a. Generate a node combination, e.g., {2, 4, 5}.These nodes are mounted with PMUs, thus observed. Save them in array O.
b. Find out all nodes adjacent to these 3 or 4 nodes.
c. Save them in array O.
5. Find out all nodes that are not in O.
a. Pick up such a node j, use rule 4 to judge if it is observed. If yes, put j in O, and pick up another node and check.
b. If all "not-in-O" nodes have been checked, compare O to the whole set G.
6. If O=G, output the node combination. That is the S-set. Quit. If O is not equal to G, generate another node combination.
7. If solution does not converge then increase the numbers in the group by one toward upper bound.
8. Output how many number of combination has been tried that is the number of measurement.
9. End.

The Integer Linear Programming is tested on IEEE 14 bus test systems. The method explained in chapter 3 has been simulated by developing a MATLAB™ program using algorithm in section 4.4and used to solve the PMU placement problem. The results obtained for complete observability of system at normal condition is shown in the Table 5.1. Fig.5.1 shows connection matrix of IEEE 14 bus system.

Table 1 Optimal PMU Location

Bus system	No. of PMU's for complete observation	Location of PMU's
IEEE 14 Bus system	3	2, 6, 9

```
**********connection matrix************
  1  1  0  0  1  0  0  0  0  0  0  0  0  0
  1  1  1  1  1  0  0  0  0  0  0  0  0  0
  0  1  1  1  0  0  0  0  0  0  0  0  0  0
  0  1  1  1  1  0  1  0  1  0  0  0  0  0
  1  1  0  1  1  1  0  0  0  0  0  0  0  0
  0  0  0  0  1  1  0  0  0  0  1  1  1  0
  0  0  0  1  0  0  1  1  1  0  0  0  0  0
  0  0  0  0  0  0  1  1  0  0  0  0  0  0
  0  0  0  1  0  0  1  0  1  1  0  0  0  1
  0  0  0  0  0  0  0  0  1  1  1  0  0  0
  0  0  0  0  0  1  0  0  0  1  1  0  0  0
  0  0  0  0  0  1  0  0  0  0  0  1  1  0
  0  0  0  0  0  1  0  0  0  0  0  1  1  1
  0  0  0  0  0  0  0  0  1  0  0  0  1  1
```

Fig 5 Connection Matrix of IEEE 14 Bus

The adjacency matrix, sometimes also called the connection matrix, of a simple labeled graph is a matrix with rows and columns labeled by graph vertices, with a 1 or 0 in position (v_i,v_j) according to whether v_i and $v_j$are adjacent or not. For an undirected graph, the adjacency matrix symmetric. Fig.5.2 shows the locations of PMU's.

```
************selected bus numbers*************
    2     6     9
```

Fig.6 Optimal Location of PMU

V. CONCLUSION

This method proposes a simple algorithm of optimalplacement of PMU's in power system for full observability of network. The OPP problem is formulated using topology based algorithm and solved using binary integer linear programming. The placementof PMU's done at normal condition. The present case also accomplished the two objectives, first to develop practical methods for determining optimal locations for PMU's and second is to develop the methods for implementation and to obtain test results. Simulation result on IEEE 14 test systems indicate that the proposed placement method satisfactorily provides observable system measurements with minimum number of PMU's.

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